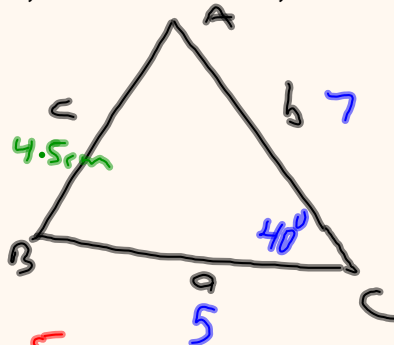


# Problems in Two Dimensions

## Solve the Triangle

Solve  $\triangle ABC$ , if  $\angle C = 40^\circ$ ,  $a = 5$  cm, and  $b = 7$  cm.



$$\frac{4.5}{\sin 40^\circ} = \frac{5}{\sin A}$$

$$5 \sin 40^\circ = 4.5 \sin A$$

$$\frac{5 \sin 40^\circ}{4.5} = \sin A$$
$$0.714 = \sin A$$

$$A = 46^\circ$$

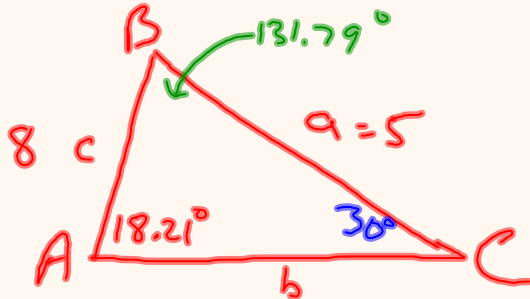
$$B = 74^\circ$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$
$$= 5^2 + 7^2 - 2(5)(7) \cos 40^\circ$$
$$c^2 = 25 + 49 - 70 \cos 40^\circ$$
$$c^2 = 74 - 53.68$$
$$c^2 = 20.4$$
$$c = \sqrt{20.4}$$
$$c = 4.5 \text{ cm}$$

# Solve a Harder Triangle

Solve  $\triangle ABC$ , if  $\angle A = 18.21^\circ$ ,  $a = 5$  cm, and  $c = 8$  cm.

In this case, there are actually two possible triangles that could be created.



$$\frac{5}{\sin 18.21} = \frac{8}{\sin C}$$

$$5 \sin C = 8 \sin 18.21$$

$$\sin C = \frac{8 \sin 18.21}{5}$$

$$\angle C = 30^\circ$$

$$\frac{b}{\sin 131.79} = \frac{5}{\sin 18.21}$$

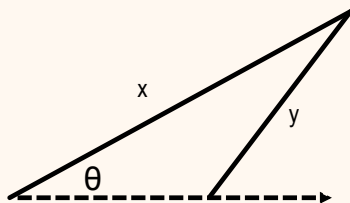
$$b = \frac{5 \sin 131.79}{\sin 18.21}$$

$$b = 11.9$$

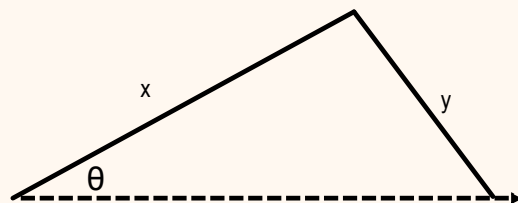
## The Ambiguous Case

We refer to the situation when two triangles are possible as the Ambiguous Case.

It only occurs if two sides, and one opposite angle are given.

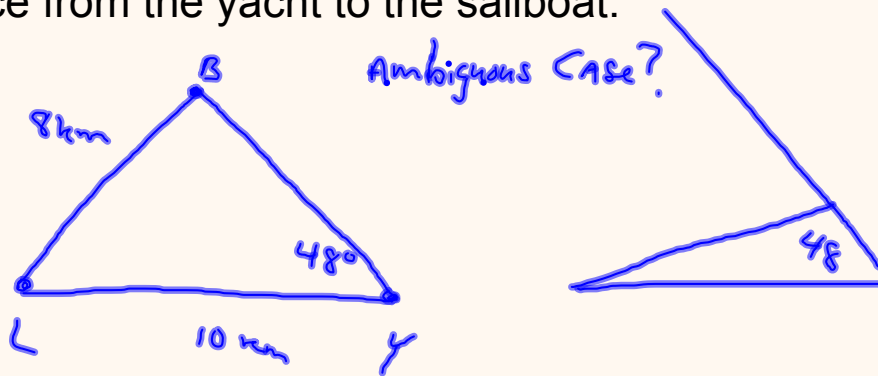


OR



## Application

A lighthouse at point L is 10 km from a yacht at point Y and 8 km from a sailboat at point B. From the yacht, the lighthouse and the sailboat are separated by an angle of  $48^\circ$ . Determine the distance from the yacht to the sailboat.



## Homework:

Solve  $\triangle ABC$  in each scenario:

- $\angle A = 58^\circ$ ,  $a = 12$ , and  $c = 10$
- $\angle A = 35^\circ$ ,  $a = 6$ , and  $b = 8$
- $\angle A = 50^\circ$ ,  $a = 10$ , and  $b = 20$
- $\angle A = 32^\circ$ ,  $a = 30$ , and  $c = 42$