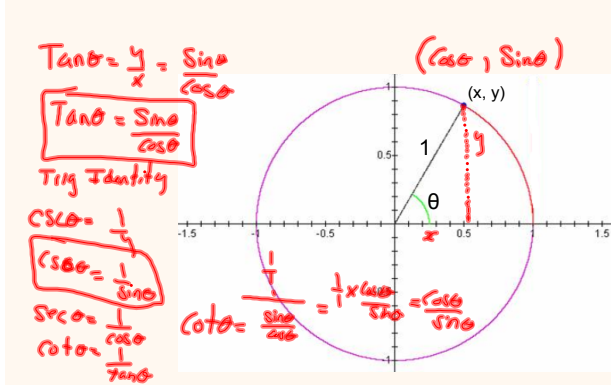


Trig Identities



Proving Identities

Prove the identity:

$$\tan \theta + \sec \theta = \frac{\sin \theta + 1}{\cos \theta}$$

Proof:

* Convert all tan or reciprocal ratios into sin, cos.

$$\begin{aligned}
 \text{LS} &= \tan \theta + \sec \theta \\
 &= \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta} \\
 &= \frac{\sin \theta + 1}{\cos \theta} \\
 &= \text{RS}
 \end{aligned}$$

Proving Identities

Prove the identity:

$$\frac{1}{\sin \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} = \csc^2 \theta$$

Proof

$$\begin{aligned}
 \text{LS} &= 1 + \frac{\cos^2 \theta}{\sin^2 \theta} \\
 &= \frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} \\
 &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta} \\
 &= \frac{1}{\sin^2 \theta} \\
 &= \csc^2 \theta \\
 &= \text{RS}
 \end{aligned}$$

Proving Identities

Prove the identity:

$$\frac{\sin \theta + \sin^2 \theta}{(\cos \theta)(1 + \sin \theta)} = \tan \theta$$

Proof:

$$\begin{aligned}
 \text{LS} &= \frac{\sin \theta + \sin^2 \theta}{(\cos \theta)(1 + \sin \theta)} \\
 \text{* Factoring is Good!} & \\
 &= \frac{\sin \theta (1 + \sin \theta)}{(\cos \theta)(1 + \sin \theta)} \\
 &= \frac{\sin \theta}{\cos \theta} \\
 &= \tan \theta \\
 &= \text{RS}
 \end{aligned}$$

The 5 Key Trig Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\sin^2\theta = 1 - \cos^2\theta$$

Proving Identities

Prove the identity:

$$\frac{1}{\cos x} - \sin x \tan x = \cos x$$

$$= \frac{1}{\cos x} - \frac{\sin x}{1} \left(\frac{\sin x}{\cos x} \right)$$

$$= \frac{1}{\cos x} - \frac{\sin^2 x}{\cos x}$$

$$= \frac{1 - \sin^2 x}{\cos x}$$

$$= \frac{\cos^2 x}{\cos x}$$

$$= \frac{(\cancel{\cos x})(\cos x)}{\cancel{\cos x}}$$

$$= \cos x$$

Strategies

Some of strategies that might work:

- Convert into sin and cos
- Factor
- Expand
- Find Common Denominators and combine fractions
- Try simplifying both the LS and RS
- Try something different
- Keep trying!

Identities to Prove:

$$1 - \cos^2\theta = \sin\theta\cos\theta\tan\theta$$

$$\frac{1}{\cos\theta} + \tan\theta = \frac{1 + \sin\theta}{\cos\theta}$$

$$\frac{\sin A}{1 - \sin^2 A} = \frac{\tan A}{\cos A}$$

$$\frac{\cos^2 x}{1 - \sin^2 x} = 1 + \sin x$$