

Reciprocal Trigonometric Ratios

All 6 Trig Ratios

We know of 3 basic trig ratios that we use frequently:

Sine

$$\sin\theta = \frac{O}{H}$$

Cosine

$$\cos\theta = \frac{A}{H}$$

Tangent

$$\tan\theta = \frac{O}{A}$$

But there are 3 other ratios that can be used:

Cosecant

$$\begin{aligned} \csc\theta &= \frac{H}{O} \\ \csc\theta &= \frac{1}{\sin\theta} \end{aligned}$$

Secant

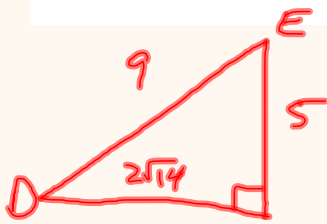
$$\begin{aligned} \sec\theta &= \frac{H}{A} \\ \sec\theta &= \frac{1}{\cos\theta} \end{aligned}$$

Cotangent

$$\begin{aligned} \cot\theta &= \frac{A}{O} \\ \cot\theta &= \frac{1}{\tan\theta} \end{aligned}$$

Example

In $\triangle DEF$, if $\csc D = \frac{9}{5}$, then determine $\cot D$



$$\begin{aligned} DF &= \sqrt{9^2 - 5^2} \\ &= \sqrt{56} \\ &= \sqrt{4 \cdot 14} \\ &= 2\sqrt{14} \end{aligned}$$

$$\csc D = \frac{9}{5}, \text{ then } \sin D = \frac{5}{9}$$

$$\cot D = \frac{A}{O}$$

$$\cot D = \frac{2\sqrt{14}}{5}$$

$$\begin{aligned} \sec E &= \frac{9}{5} \\ \cot E &= \frac{5}{2\sqrt{14}} \end{aligned}$$

Example

Each angle is in the first quadrant. Determine the measure of each angle, to the nearest degree.

a) $\csc A = 8$

$$\sin A = \frac{1}{8}$$

$$A = \sin^{-1}\left(\frac{1}{8}\right)$$

$$A = 7^\circ$$

b) $\sec B = \frac{5}{2}$

$$B = 66^\circ$$

c) $\cot C = \frac{5}{16}$

$$C = 73^\circ$$

d) $\csc \theta = 2$

$$\theta = 30^\circ$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = 30^\circ$$

Example

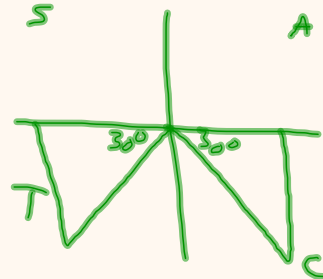
Determine two angles between 0° and 360° that have a cosecant of -2 .

$$\csc \theta = -2$$

$$\sin \theta = -\frac{1}{2}$$

$$\begin{aligned}\theta &= 180 + 30^\circ \\ &= 210^\circ\end{aligned}$$

$$\begin{aligned}\theta &= 360 - 30^\circ \\ &= 330^\circ\end{aligned}$$



Example

Simplify:

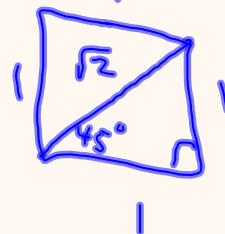
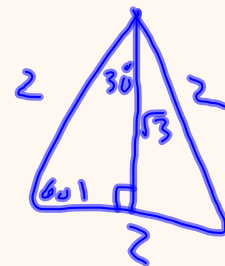
$$\sec 60^\circ + \csc 30^\circ$$

$$= \frac{1}{\cos 60^\circ} + \frac{1}{\sin 30^\circ}$$

$$= \frac{1}{\frac{1}{2}} + \frac{1}{\frac{1}{2}}$$

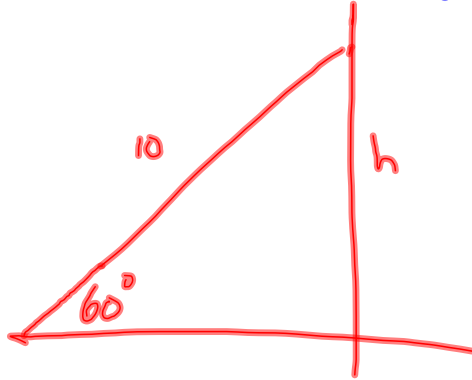
$$= 2 + 2$$

$$= 4$$



Example:

A 10 m ladder leans against a wall. It makes a 60° angle with the floor. How high up does it reach?



$$\begin{aligned}\sin 60 &= \frac{h}{10} \\ 10 \sin 60 &= h \\ 10 \left(\frac{\sqrt{3}}{2} \right) &= h\end{aligned}$$

$$h = 5\sqrt{3} \text{ m}$$

